Fatigue Life Prediction Using Hybrid Prognosis for Structural Health Monitoring

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A reliable prognostics framework is essential to address failure mode mitigation and life cycle cost of aerospace systems. Metallic aircraft components are subject to a variety of in-service loading conditions and prediction of fatigue life remains a critical challenge. A hybrid prognostic model, which can accurately predict the crack growth regime and the residual useful life estimate (RULE) of aluminum components, is developed to address this issue. This model uses an integrated technique; coupling physics based approach with a data-driven approach. Different types of loading such as constant amplitude, random and overload are considered and the developed methodology is validated with the experimental data available in the literature. The results indicate that fusing the measured data and physics based models improves the accuracy of prediction compared to a pure data-driven or physics based approach.

I. Introduction

Accurate estimation of fatigue life of metallic components under complex loading conditions is critical to the safety and reliability of aerospace vehicles. A majority of the current available fatigue life prediction models are deficient in predicting damage under random or flight profile service loads. The inherent accuracy is due to the stochastic nature of crack propagation in metallic structure. A significant amount of work has been reported on the development of reliable prognostic frameworks. Research is primarily focused on data driven approaches, however physics based models have also been utilized, the details of which will be presented in the subsequent discussion. Data driven approaches do not use any physics based information, and hence the prediction accuracy is inferior to those based on physics when there is sparse data or incomplete knowledge. Physics based approaches have difficulty adapting to variations in response as a result of material scatter, environmental changes, and other unclassifiable but significant noises. Thus, a hybrid prognostic model, which uses a fusion of physics based model information and measured data, is expected to provide more accurate and reliable information on damage prognosis and residual useful life estimation (RULE).

Ling et al.¹ proposed a method for the integration of structural health monitoring (SHM) with fatigue damage prognosis. The prognosis methodology uses a fracture mechanics based crack growth model, focusing on predictions under uncertainties in the data and model errors using Wheeler’s retardation model. Ling et al.² also presented a method for predicting under stochastic loading by investigating techniques such as rainflow counting, Markov chain method and autoregressive moving average (ARMA) model. Sankararaman et al.³ presented a methodology for prognosis under variable amplitude multiaxial loading, where an equivalent stress intensity factor as a function of the crack length and the loading is used. Then, a Gaussian process regression is used to calculate the stress intensity factor at any crack length. Mohanty et al.⁴ developed a Gaussian process based prognosis framework, combining on-line and off-line information, for damage state estimation and RULE of structural hotspots under complex loading such as random and FALSTAFF¹⁹,²⁰ (Fighter Aircraft Loading Standard for Fatigue) Although this model can be highly accurate, the accuracy of prediction is dependent on the available training data. In the initial stage, where there is less training data, the prediction is not accurate. The prediction accuracy increases over time as more training data becomes available⁵. Several other methods⁶–¹⁴ have been proposed for fatigue crack growth modeling. The difference between these models is not very significant. These models depend on the geometry of the specimen and therefore have limited applications.

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In this paper, we present a hybrid prognosis methodology which uses the physics based model information integrated with a data driven approach for damage state prediction and RULE. The results are validated with available experimental data. This model will later be integrated with the detection scheme being developed in-house, to develop a fully integrated SHM framework.

II. Hybrid Prognosis Theory

The fatigue crack growth behavior for a given specimen can be predicted by combining knowledge of the underlying mechanics of crack growth and future loading. The hybrid prognosis framework presented in this paper considers only elementary physics models whose behaviors are inferred and updated using data driven approaches. The combination of physics and data driven approaches allows for the consideration of proper fracture mechanisms while correcting for material variations and uncertainty in the model parameters by using data driven model updating. Thus, although simple physics models are used, the accuracy of the hybrid framework is greater than those of data driven and physics based models alone, as shown in the results presented in a later section.

Linear elastic fracture mechanics and most fracture theories state that the crack growth rate ($da/dN$) is a function of the stress intensity factor ($\Delta K$), SIF, as shown in Eq. 1

$$\frac{da}{dN} = f(\Delta K)$$

Due to the exponential nature of crack growth, typically models describe the relationship between crack growth rate and SIF use log-log transforms. Figure 1 illustrates a commonly observed trend showing three critical zones, Stages I-III, of crack growth. Typically, prognosis algorithms are applied during Stage II or sub-critical crack growth and used to predict ultimate fracture. In some cases, for example constant amplitude loading, this regime is linear. Models such as Paris’ Law are well suited to capture this behavior. For cases such as overloads and under-loads, this regime can be highly nonlinear and discontinuous requiring the use of advanced physics models, which are often unavailable. In the hybrid prognosis framework presented, the exact relationship between crack growth rate and SIF for a given cycle is inferred from the available data based on an assumption of a linear relationship with non-constant coefficients in log space. The constants of the linear fit are a function of historical crack growth data, future loading (i.e. overloads/under-loads), basic material properties and cycles, continuously evolving and adapting as more data is available. This is shown in Eq. 2, where $C_1$ and $C_2$ are coefficient functions, $M$ denotes material parameters, $P$ represents loading, and subscripts N-1 and N+1 denote previous and future loading cycles.

$$\log\frac{da}{dN} = C_1(a_{N-1}, M, P_{N,N+1}, N) + C_2(a_{N-1}, M, P_{N,N+1}, N)\log(\Delta K)$$

Initial estimates for these parameters (i.e., prior to data acquisition) can be obtained through the basic material constants used in Paris’ law. This reduces the crack growth rate estimation to a classical Paris’ law extrapolation. However as the SHM framework provides data on crack length and locations, as well as load monitoring and cycle counting, the constants are updated, allowing them to model and capture the nonlinear and discontinuous behavior. In order to predict the fatigue crack growth of a specimen, Eq. 2 needs to be formulated in terms of measurable parameters and integrated until ultimate fracture. Therefore the parameters in Eq. 2 must be written in terms of these data, this requires relating SIF to known or quantifiable parameters.

We will assume that the SIF can be expressed as a general function

$$\Delta K_N = f(a_N, P_N, S)$$

For simple structures, analytical expressions of SIF are available, describing its dependence on geometry, crack length and applied load. When an analytical expression is available, this expression can then be directly substituted into Eq. 1 and the future crack growth solved. However, in the absence of this information numerical methods must be utilized to provide estimates of SIF. Either method is
acceptable and suitable for use in the proposed framework.

To predict RULE, Eq. 2 is numerically integrated until the crack growth rate becomes unstable. Prior to integration, the non-constant coefficients, $C_1$ and $C_2$, must be determined. To calculate these coefficients a least squares regression algorithm is used. The training data for the algorithm is heterogeneous in nature originating from multiple sources. Although only in-situ measured data is necessary to determine $C_1$ and $C_2$, introduction of additional data from previous experiments, expert knowledge, Paris’ Law coefficients, advanced and multi-scale models can drastically improve the results. The developed hybrid prognosis model can be applied at the first instance of crack initiation or at any measured point in time. The framework is numerically efficient and suitable for real time applications.

III. Results

A. Constant amplitude loading

The efficiency of the hybrid prognosis model is illustrated using a compact tension (CT) test article with available test data. Wu and Ni\textsuperscript{15} conducted 30 constant amplitude fatigue tests on the CT samples to generate a statistically large dataset and concluded that the results were log-normally distributed. CT specimens, 50mm wide and 12mm thick, were fabricated from 2024-T351 aluminum alloy according to the ASTM standard E647-93\textsuperscript{16}. The specimen were pre-cracked up to 15 mm and extended to 18mm of crack length. A constant amplitude load, with maximum amplitude of 4.5 kN and minimum amplitude of 0.9 kN, was applied during both the pre-cracking and fatigue tests. The crack lengths were measured using images taken by a microscope, until the specimen fractured.

An analytical expression for the stress intensity factor (SIF), $K$, as a function of the crack length ($a$), geometry and load for a CT specimen was used to solve the differential equation shown in Eq. 2. This is shown in Eq. 4 and the variables are defined in Fig. 2.

$$K_{\text{max}} = \frac{P_{\text{max}}}{B} \sqrt{\frac{\pi}{W}} \left[16.7(a/W)^{1/2} - 104.7(a/W)^{3/2} + 369.9(a/W)^{5/2} - 573.8(a/W)^{7/2} + 360.5(a/W)^{9/2}\right]$$

$$K_{\text{min}} = \frac{P_{\text{min}}}{B} \sqrt{\frac{\pi}{W}} \left[16.7(a/W)^{1/2} - 104.7(a/W)^{3/2} + 369.9(a/W)^{5/2} - 573.8(a/W)^{7/2} + 360.5(a/W)^{9/2}\right]$$

$$\Delta K = K_{\text{max}} - K_{\text{min}}$$

where, $P_{\text{max}}$ is the maximum amplitude and $P_{\text{min}}$ is the minimum amplitude of the cyclic loading. To start the prediction at a given cycle, first the non-constant coefficients must be determined. This was achieved through linear fitting of all acquired data points (i.e. all known $da/dN$ and $\Delta K$) with two additional training data points. In order to regress the data, it must be transformed from crack length versus cycles to crack growth rate versus SIF. Numerical differentiation and Eq. 4 were used to accomplish this. The training data was derived from Paris’ Law coefficients using the average response of 30 specimen dataset. The Paris’ Law coefficients were used to evaluate the crack growth rate and SIF at the approximate beginning and end of the Stage II crack growth, points that are far away from the measured dataset. As the crack length increased and more data were available, the weight of the training data is reduced, relying primarily on the measured data. Once the coefficients are evaluated, the crack growth rate in Eq. 2 can be discretely integrated until RULE has been reach. Considering a very small number of cycles ($\Delta N$), the crack length at $(N + \Delta N)$ cycles can be obtained as,

$$a_{N+\Delta N} = a_N + \frac{da}{dN} \Delta N$$

where $a_N$ is the crack length after $N$ cycles.
The methodology was applied to several starting points to demonstrate the convergence of RULE as more data is available. A random dataset was chosen to validate the hybrid prognosis framework. Figure 3 shows the experimental crack length dataset and initial prediction as well as the data transformed into prediction space. The RULE and error in RULE are also plotted in the same figure. The error is defined by \( \frac{RULE - RULE_{predicted}}{RULE} \).

Figure 4 shows the cumulative results of several predictions superimposed on Fig. 3.

The experimentally measured data set is plotted along with an initial prediction starting at 8000 cycles. The initial slope of the da/dN vs SIF matches well with experimental data; approximately 11% over prediction.
The prior results utilize all available data to determine the non-constant coefficients of Eq. 2. However, by reducing the amount of data used for regression, the algorithm is more suited to adapt to small changes in crack growth rates. To illustrate this, only the last five available data points were used to regress Eq. 2. Figure 5 shows the prediction starting at the tenth available data point and results in a fail-safe prediction, i.e. $\text{RUL}_{\text{predicted}} < \text{RUL}$. Fig. 6 shows several more predictions superimposed onto Fig. 5.

![Graphs showing crack length, crack speed, and error in total fatigue life vs. cycles.](image)

Fig. 5. The experimentally measured data set is plotted along with the initial prediction starting at 18000 cycles and using only previous 5 data points. The initial slope of $\frac{da}{dn} \text{ vs } \Delta K$ matches well with the experimental data with an error of 5%.

![Graphs showing crack length, crack speed, and error in total fatigue life vs. cycles.](image)

Fig. 6. Multiple predictions are made using only the previous five data points at every iteration. The results improve as the training data increases, and the error reduces to within 2% towards the end.
B. Random amplitude loading

Wu and Ni\textsuperscript{15} published a data set for the random loading. The specimens used were the same as the one used for the constant amplitude-loading test. A band-limited (5-15 Hz) and uniformly distributed power spectrum density function was used to generate random signals with a mean value of 5 kN and a random amplitude of mean 1.118 kN and a standard deviation of 0.552 kN.

The SIF calculation and training methodology was identical to that of the constant loading. However, the training data was modified based on the mean values of the random dataset. The results for multiple predictions are shown in Fig. 7. Predictions at higher data points results in nearly zero error, therefore were not included in the plot.

C. Overload:

McMaster and Smith\textsuperscript{17} presented a data set for a center-cracked specimen made from an Al2024-T351 alloy under an overload. The specimen was 100mm wide, 250 mm long and 14mm thick. The overload test consisted of three overload excursions applied at crack length intervals of $2a/W = 0.4, 0.5, 0.6$. A hole of 4mm was made at the center of the specimen, followed by electro discharge machining of a starter notch of length 2mm and height 0.2mm. The geometry of the test article is shown in Fig. 8.

The stress intensity factor for the center-cracked plate\textsuperscript{18} is calculated as,

![Fig. 7 Multiple predictions are made under random amplitude loading starting at 50000 cycles and using only previous five data points at every iteration. The accuracy is well within 2%.

![Fig. 8 Plate with a central hole\textsuperscript{16} under constant amplitude loading with overload at specific intervals]
where, $a$ is the crack length, $F$ is the geometric factor, $\sigma$ is the magnitude of applied loading.

To capture the crack closure phenomenon associated with overloads, the algorithm for determining the non-constant coefficients was modified. Typical overload behavior in the log-log plot of crack growth rate versus SIF is illustrated in Fig. 9. A linear growth rate is observed in stage II; however, once the specimen has been overloaded the crack closure phenomenon reduces the growth rate slope significantly. This new behavior continues until with the original linear response. Although Fig. 9 shows the overload slope as linear, it has been observed to be highly nonlinear. However, since the hybrid prognosis model adapts to new data, an initial linear model is more than adequate to yield good predictions. For this sample, training data used to calculate the slopes of the overload region were averaged from the experimental data.

In order to consider overloads in RULE, the times at which the overload excursions must be either assumed or modeled. To make the prediction, the cycles at which the experimental overloads occurred were assumed to be known, similar to an “oracle” approach. These results are shown in Fig. 7.

However, if the number of overloads and when they occur are unknown (as in most problems), the prediction results can vary and are strongly dependent on when the overloads occur. For example, the same specimen was simulated with four overloads occurring at random cycles. This is shown in Fig. 11. The results show prediction capabilities within 5% for randomized future loading.

\[
K = F \sigma \sqrt{\pi a}
\]

\[
F = \varphi \psi
\]

\[
\alpha = \frac{a}{W}, \lambda = \frac{\pi}{2}\alpha, \delta = \frac{a}{R}, \gamma = \frac{R}{W}, \beta = \frac{a - \gamma}{1 - \gamma}
\]

\[
\varphi = \sqrt{\frac{1}{3} (\tan \lambda + \sin 2\lambda) \left( 1 + \frac{\varepsilon^2 (2 - \varepsilon)}{1 - \varepsilon} \right)} - \sqrt{1 + 2g}
\]

\[
g = 0.13 \left( \frac{2}{\pi} \arctan (\delta) \right)^2
\]

\[
\varepsilon = \frac{2}{\pi} \arctan \left( 0.6 \sqrt{\delta} \right)
\]

\[
\psi = \xi \left( 3 \beta^2 - 2 \sqrt{\beta} \right)
\]

\[
P = \log \left( \frac{\xi^2}{\psi} \right)
\]

\[
\beta^* = \frac{y \delta}{\sqrt{2(2y - 1) \xi^2}}
\]

\[
\xi = 1 + \frac{2}{\pi} \arctan (1.5 \sqrt{\delta})
\]
Fig. 10 Multiple predictions are made with known overload data. The results predict very well with-in an error of 5% as the overload point is known

Fig. 11 Multiple predictions are made with unknown overload data. The predictions do not match the exact overload data but the prediction error is well within an error of 5%.

IV. Conclusions

A hybrid prognosis methodology is developed integrating simple physics based approach with experimental data. The algorithm provides high fidelity predictions of RULE for CT specimens subject to various loading conditions. The RULE prediction is within ±5% of the actual RULE for constant amplitude loading. The methodology was applied to random loading conditions; the mean of the random data set is used as the initial training data. The RULE prediction was within ±5% for the random loading case, reducing to ±2% as the training data increased. The algorithm was modified to incorporate the crack closure phenomenon observed during an
overload. Once again, an error of ±5% was observed if we assume that the point of overloads is known. However, even if the overload points are unknown, the algorithm was still able to predict within 5% error.

Acknowledgements

This research is supported by the U.S. Department of Defense, U.S. Air Force Office of Scientific Research Multidisciplinary University Research Initiation Grant, FA95550-06-1-0309, Technical Monitor David Stargel.

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